What is BELL's Inequality?

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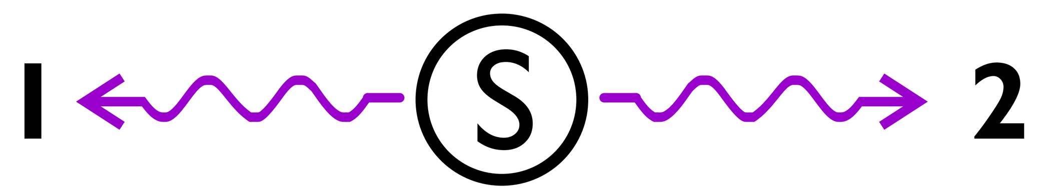


In 1935 EINSTEIN, PODOLSKY and ROSEN discussed one of the problems of quantum mechanics: The superposition principle, which allows two distant objects to be in a joint quantum state, seems to require a 'spooky action at a distance' to establish measurement results. Such a superposition state can be described as:

$$|\Psi\rangle^{-} = \frac{1}{\sqrt{2}}(|\uparrow_1, \rightarrow_2\rangle - |\rightarrow_1, \uparrow_2\rangle)$$
$$= \frac{1}{\sqrt{2}}(|\nearrow_1, \nwarrow_2\rangle - |\nwarrow_1, \nearrow_2\rangle)$$

Quantum mechanics requires that measurement results are not only correlated in H/V, but also in +/- 45°. However, a +/- 45° state has no defined outcome in a H/V measurement and leads to a random result. This also holds if the two photons are very far apart.

In 1964 JOHN BELL realized that he could describe this 'non-locality' in a simple way with correlations. By measuring the detector clicks, you can obtain a correlation:

$$E = \frac{N_{12} + N_{1'2'} - N_{1'2} - N_{12'}}{N_{12} + N_{1'2'} + N_{1'2} + N_{12'}}$$

It can be shown that for an underlying classical model, a combination 'S' of correlations in different bases will be between -2 and 2. This is BELL's inequality.

$$S := E(\underset{=}{a}, b) - E(a, b') + E(a', b) + E(a', b')$$

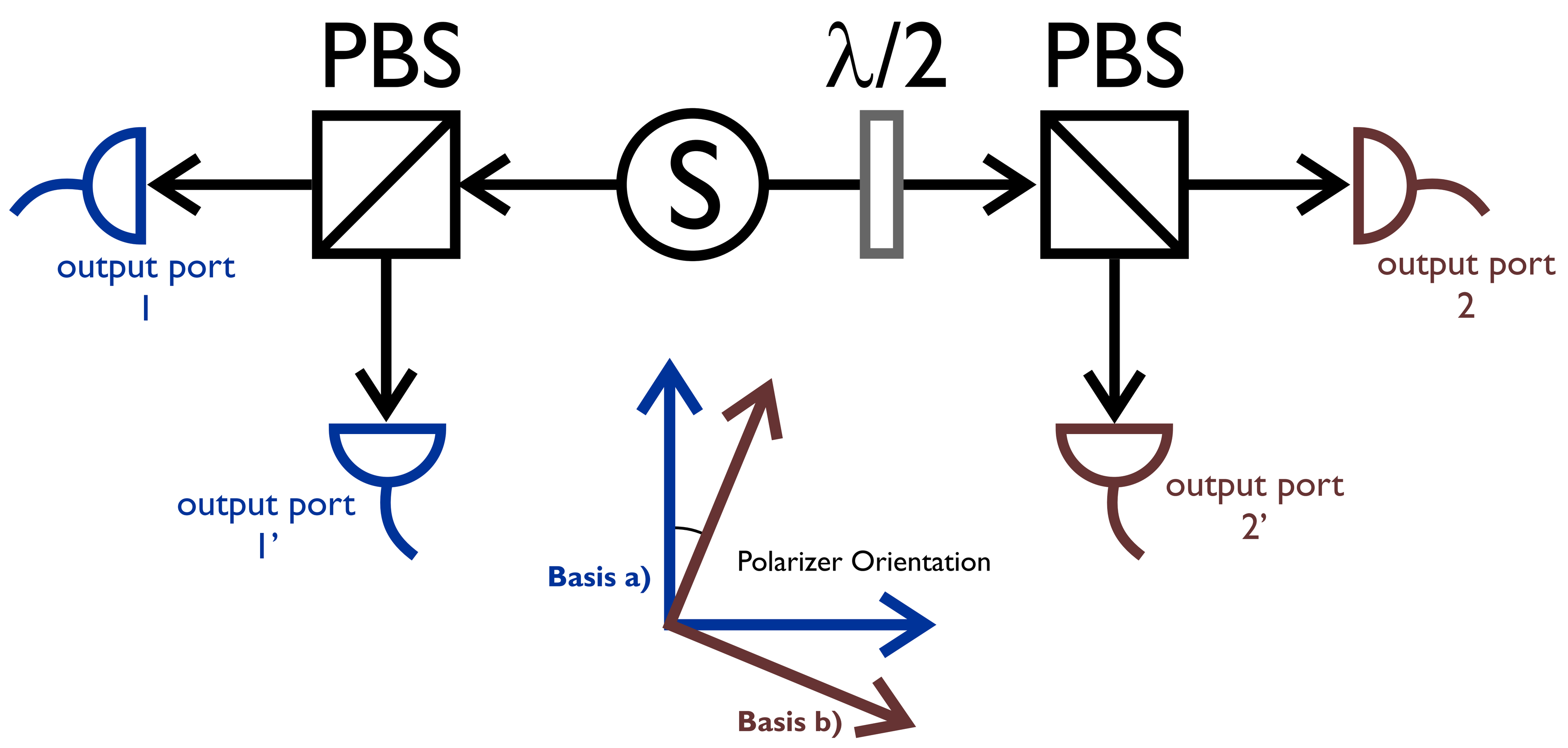
one side in basis a
other side in basis b

$$-2 \leq S \leq 2$$

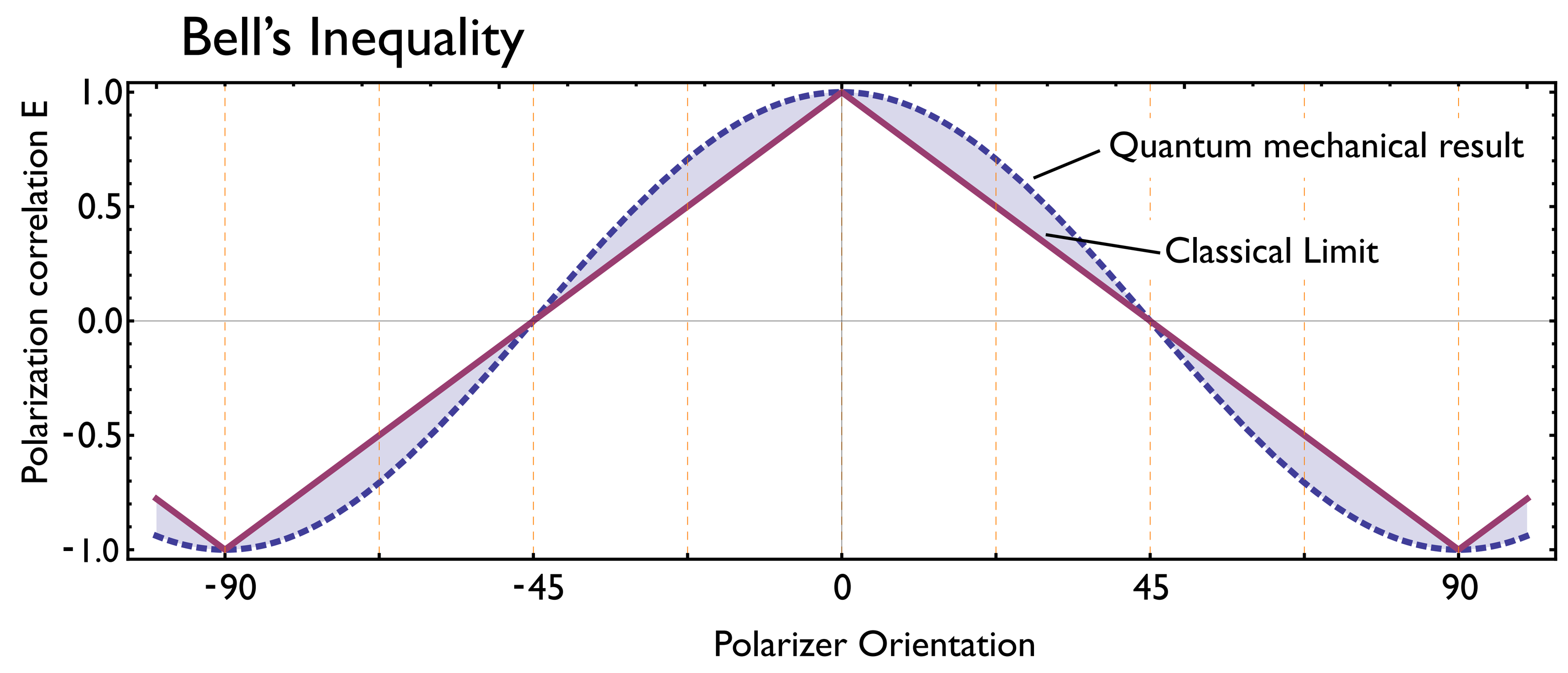
On the other hand it is possible to show that for an 'entangled state' quantum mechanics predicts a value up to S=2.828, following MALUS' law:

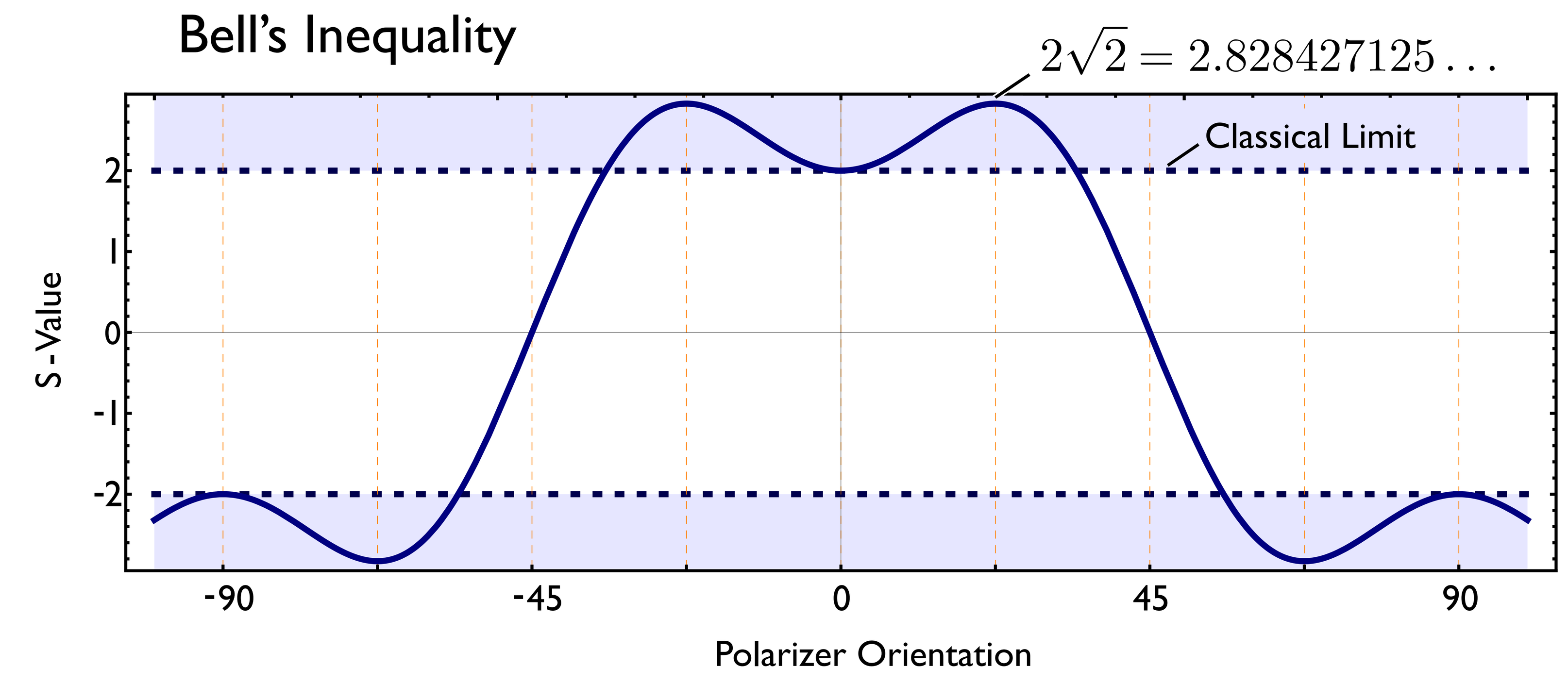
$$\cos(a,b) - \cos(a,b') + \cos(a',b) + \cos(a',b')$$

In 1982 ALAIN ASPECT and PHILLIPE GRANGIER measured the properties on this joint quantum state. They found that indeed BELL's Inequality is violated, resulting in values above 2.0. One experimental indication that there is no classical model to explain this correlation.



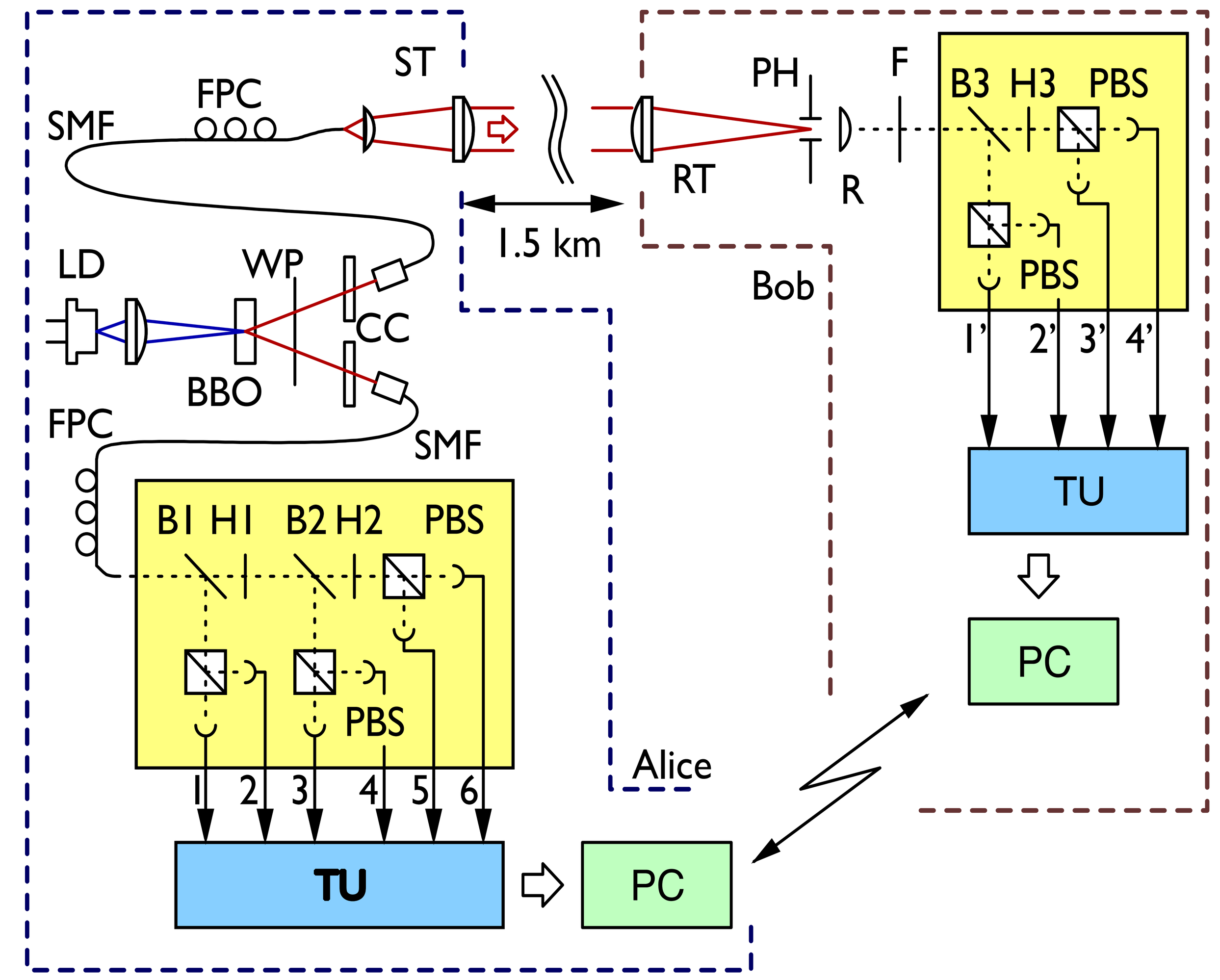
S: source of entangled photons
λ/2 plate: rotates the polarization by an angle
PBS: polarizing beam splitter, measures the polarization state



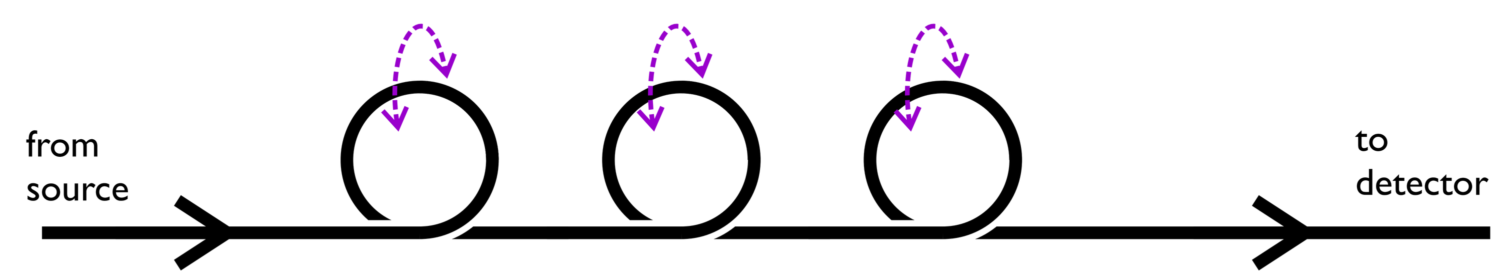


How can BELL's Inequality be used for Quantum Information:

Degree of violation measures knowledge of an eavesdropper (presence of hidden variables)



Rotating the measurement basis with an optical fiber



For a measurement on the joint quantum state it is important to have a suitable relative orientation of the polarization measurements.

By bending and twisting a fiber it is possible to change the relative measurement orientation.

If you reach quickly values above 2.5 you should consider doing a PhD in the field of quantum information. See our website below to apply.

Centre for Quantum Technologies, Singapore

http://quantumlah.org